

## MATHEMATICS

### ALGEBRA

1. Don and Jerry are fishing. They each have several fishing poles, and each pole has several worms on its line. Don's poles each have 6 worms on their lines. Jerry's poles each have 11 worms on their lines. Between the two of them, Don and Jerry have 13 poles and 103 worms. How many poles does Don have?

A) 10  
B) 8  
C) 6  
D) 5  
E) 4

2. The real numbers  $a$  and  $b$  which are different from 0 satisfying the condition  $ab = a - b$ . What is the value of  $\frac{a}{b} + \frac{b}{a} - ab$

A)  $-\frac{1}{2}$   
B) 2  
C) -2  
D)  $\frac{1}{2}$   
E)  $\frac{1}{3}$

3. Given that  $x = \frac{1}{\sqrt{3}-\sqrt{2}}$ ,  $y = \frac{1}{\sqrt{3}+\sqrt{2}}$ . What is  $3x^2 - 5xy + y^2$ ?

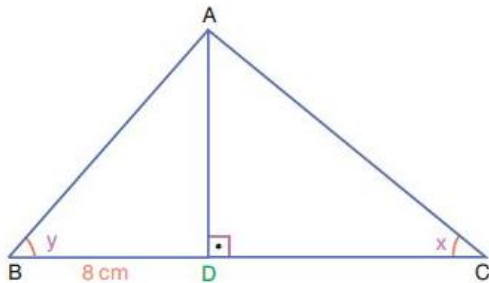
A)  $15 + 2\sqrt{6}$   
B)  $15 - 2\sqrt{6}$   
C)  $10 - 2\sqrt{6}$   
D)  $15 + 4\sqrt{6}$   
E)  $10 - 4\sqrt{6}$

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4. Let  $a, b$  be integers; if  $a^2$  is odd and  $b^3$  is even, then which of the followings is odd?

A)  $2a^3 + b^2$   
B)  $6a^5 + b^4$   
C)  $a^3 + b^2$   
D)  $2a + 3b$   
E)  $3a + 5b$

5. In the figure below,  $ABC$  is a triangle with  $AD \perp BC$ ,  $|BD| = 8\text{cm}$ ,  $\cos y = \frac{4}{5}$  and  $\sin x = \frac{2}{3}$ . What is  $|DC|$ ?



- A)  $2\sqrt{5}$   
 B)  $5\sqrt{5}$   
 C)  $6\sqrt{5}$   
 D)  $4\sqrt{5}$   
 E)  $3\sqrt{5}$
6. Mr. Kenzi is a student teacher at the local junior high. His first period class of 40 students averaged 96% on a recent test. His second period class of 20 students averaged a 90% on the same test. What was combined average for both classes?

- A) 86  
 B) 88  
 C) 90  
 D) 92  
 E) 94

7.  $(\log_2 125 + \log_4 25 + \log_8 5) (\log_{125} 8 + \log_{25} 4 + \log_5 2) = ?$

- A) 10  
 B) 11  
 C) 12  
 D) 13  
 E) 14

8. If  $A = \frac{21}{19} + \frac{11}{29}$ , then  $\frac{18}{29} - \frac{2}{19}$  equals:

- A)  $2 - A$   
 B)  $1 - A$   
 C)  $A$   
 D)  $A + 1$   
 E)  $A + 2$

9.  $99 - 97 + 95 - 93 + 91 - 89 + \dots + 7 - 5 + 3 - 1$  equals:

- A) 100
- B) 50
- C) 75
- D) 25
- E) 10

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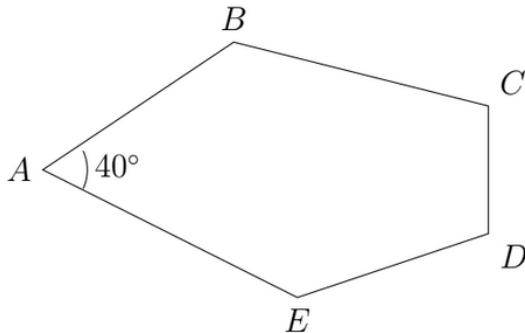
10. In a certain population, 40 percent of all people have biological characteristic X; the others do not.

If 8000 people have characteristic X, how many people do not have X?.

- A) 12000
- B) 7200
- C) 4800
- D) 20000
- E) 96000

## GEOMETRY

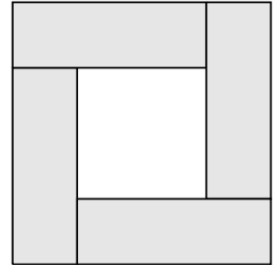
11. In  $ABCDE$ ,  $\angle A = 40^\circ$ ,  $\angle B = \angle E$  and  $\angle C = \angle D$ . What is the sum of  $\angle B$  and  $\angle C$ ?



- A)  $260^\circ$   
 B)  $250^\circ$   
 C)  $240^\circ$   
 D)  $230^\circ$   
 E)  $225^\circ$
12. Two angles of an isosceles triangle measures  $80^\circ$  and  $x^\circ$ . What is the sum of possible values of  $x$ ?

- A)  $60^\circ$   
 B)  $70^\circ$   
 C)  $80^\circ$   
 D)  $90^\circ$   
 E)  $100^\circ$

13. The diagram shows four identical rectangles placed inside a square. The perimeter of each rectangle is 24 cm. What is the perimeter of the large square?

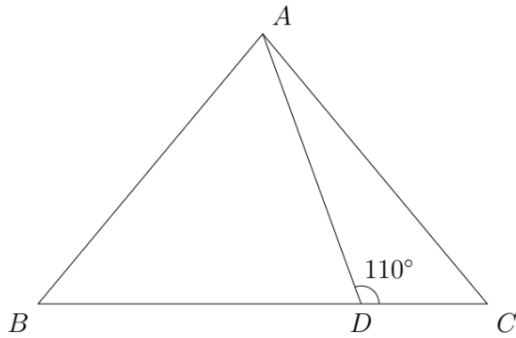


- A) 24 cm  
 B) 30 cm  
 C) 36 cm  
 D) 48 cm  
 E) 56 cm

14. The ratio of the side lengths of two similar triangles is  $5 : 4$ , and perimeter of the greater triangle is 30 cm. What is length of the shortest side of the smaller triangle, if its side lengths are consecutive even numbers?

- A) 10 cm  
 B) 8 cm  
 C) 6 cm  
 D) 4 cm  
 E) 2 cm

15.  $ABC$  is an isosceles triangle with  $AB = AC$ . Point  $D$  on  $BC$  side such that  $3\angle CAD = \angle BAD$ . If  $\angle ADC = 110^\circ$ , what is the  $\angle ABC$ ?



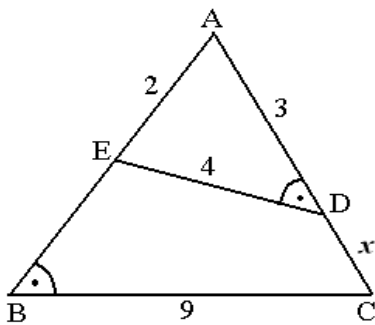
- A)  $20^\circ$
- B)  $40^\circ$
- C)  $50^\circ$
- D)  $60^\circ$
- E)  $80^\circ$

17. If sides of a square increase 1 m. then its area increases  $101 \text{ m}^2$ . What is the length of the original square's sides?

- A) 20
- B) 30
- C) 50
- D) 75
- E) 100

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16. In the given figure,  $\angle ABC = \angle EDA$ ,  $AE = 2\text{cm}$ ,  $AD = 3\text{cm}$ ,  $ED = 4 \text{ cm}$  and  $BC = 9\text{cm}$ , what is  $DC$ ?

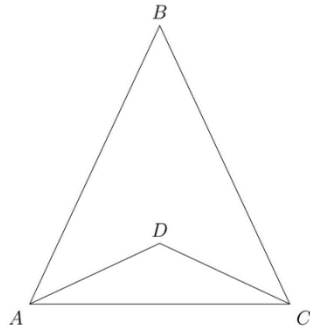


- A) 1
- B) 1.5
- C) 2
- D) 2.5
- E) 3

18. The vertices of a triangle are  $(1, 1)$ ,  $(5, 4)$  and  $(1, 4)$ . Find the area of the triangle.

- A) 7
- B) 8
- C) 9
- D) 6
- E) 11

19.  $ABC$  is an isosceles triangle with  $|AB| = |BC|$ .  $D$  is a point inside  $\triangle ABC$  such that  $|AD| = |DC|$ . If  $\angle ABC = 50^\circ$  and  $\angle ADC = 130^\circ$ , then  $\angle BAD = ?$



- A)  $35^\circ$
- B)  $40^\circ$
- C)  $45^\circ$
- D)  $50^\circ$
- E)  $55^\circ$

20.  $P$  is a point inside square  $ABCD$ . Obtuse angle of isosceles triangle  $DPC$  is  $150^\circ$ . What is the size of angle  $DAP$ ?

- A)  $30^\circ$
- B)  $15^\circ$
- C)  $45^\circ$
- D)  $60^\circ$
- E)  $35^\circ$

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## COMBINATORICS

21. 29 people chated in WhatsApp. Each person chated to exactly 10 people. What was the total number of chats?

- A) 290
- B) 280
- C) 145
- D) 135
- E) 130

22. A family consists of a mother, father, 2 girls, and 3 boys. How many different ways can the family choose one girl to wash the dishes and one boy to dry the dishes?

- A) 6
- B) 7
- C) 8
- D) 9
- E) 10

23. The Labanese restaurant offers the following side order choices: tater tots, french fries, and onion rings. They come in two sizes: small and large. How many possible combinations are there?

- A) 5
- B) 6
- C) 7
- D) 8
- E) 9

24. Judy wakes up in the morning and finds that she has three pairs of shorts and four shirts. How many ways can she dress for school?

- A) 5
- B) 6
- C) 9
- D) 12
- E) 14

25. You are at the movie theatre to watch Doctor Strange with your friends. You have enough money to buy popcorn or chips. There are three sizes of popcorn: small, medium, and large. And there are three kinds of chips: cheese, chili cheese, or no cheese. How many movie snack options do you have?

- A) 6
- B) 7
- C) 8
- D) 9
- E) 10

- A) 6
- B) 7
- C) 8
- D) 9
- E) 10

26. Mary goes to the library to borrow some books. There are 3 picture books, 6 fictional books, and 4 non-fictional books she is interested in reading. How many books can she read?

- A) 6
- B) 9
- C) 10
- D) 12
- E) 13

28. Margeret fills in 2X2 table using numbers 1,2,3 and 4 each once. One number is placed in an unit square exactly once. She places 4 as shown in the figure. In how many ways can she place the remaining numbers?

	4

- A) 5
- B) 6
- C) 7
- D) 8
- E) 9

27. How many 3-letter words can you make using letters a and b only? (e.g: aab,no need to have meaning)

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29. Ally builds a cube by gluing together 8 smaller cubes, each of the same size, with 2 smaller cubes along each edge. He then paints opposite faces of built cube with same color, using colors red, blue and yellow. How many of smaller cubes have faces with two yellow faces?

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

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30. Five same size cakes will be distributed among six children equal in size. Into how many equal size should cut each of these cakes?

- |      |      |      |
|------|------|------|
| A) 2 | B) 3 | C) 6 |
| D) 5 | E) 4 |      |

## NUMBER THEORY

31. Let  $p$  and  $q$  be prime numbers.  $p + q^2$  is a square of any positive integer. What value of  $p$  does not satisfy the rule?

- A) 3
- B) 5
- C) 7
- D) 8
- E) 11

32. In a classroom there are only chairs with four legs. All children are seated except one child who is standing. If there are 62 legs totally, then what is the number of children?

- A) 6
- B) 7
- C) 8
- D) 9
- E) 10

33. How many digits are there for a number  $(102018 + 1)^2$ .

- A) 2018
- B) 4037
- C) 4036
- D) 2019
- E) 4038

34. Least common multiple of three prime numbers is 105. What is GCD of three numbers which are one less than the prime numbers?

- A) 1
- B) 3
- C) 2
- D) 4
- E) 6

35. Difference of two integers is 9. If one of them is divisible by 6 then which one of the following is true?

- A) the other is divisible by 6,too.
- B) the other is divisible by 9.
- C) the other is divisible by 2
- D) the other is divisible by 3
- E) the other is divisible by 4

37. What is the smallest positive 3-digit integer, with all different digits, that is divisible by each of its non-zero digits?

- A) 101
- B) 100
- C) 102
- D) 103
- E) 104

36. I am 13 years old, and my coach is 31 years old, which is my age with the digits reversed. What is the fewest number of years in which the digits of our ages will be reversed again?

- A) 11
- B) 10
- C) 9
- D) 12
- E) 20

38. For what value of  $n$  is the four-digit number  $712n$ , with unit's digit  $n$ , divisible by 18?

- A) 4
- B) 6
- C) 8
- D) 2
- E) 0

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39. What is the last digit of the multiplication  
 $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times \dots \times 19 \times 20$

- A) 0
- B) 2
- C) 5
- D) 7
- E) 9

40. Let  $xy$  and  $yx$  be two two-digit numbers such  
that  $xy + yx = 44$ .

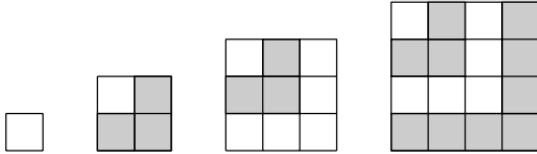
Which one of the following is  $x + y$ ?

- A) 6      B) 11      C) 5
- D) 4      E) 3

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**LOGIC**

41. Based on the pattern, how many more shaded squares than unshaded squares will be in the 50th diagram in the sequence.



- A) 50
- B) 60
- C) 80
- D) 100
- E) 150

42. The big cube is made up 64 white small cubes. All the faces of the big cube are then painted in red. How many of the small cubes have exactly two painted red faces?

- A) 28
- B) 20
- C) 12
- D) 10
- E) None of the above

43. A book has 500 pages numbered 1, 2, 3, and so on. How many times does the digit 1 appear in the page numbers?

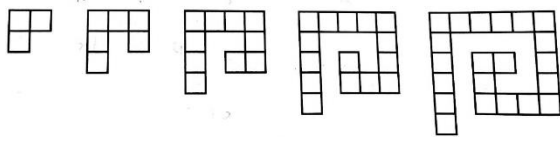
- A) 150
- B) 160
- C) 180
- D) 190
- E) 200

44. What number is missing at the end of series 3, 11, 19, 27, ?

- A) 35
- B) 36
- C) 37
- D) 38
- E) 39

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45. In the 30<sup>th</sup> picture , how many squares are there ?



- A) 465
- B) 475
- C) 486
- D) 496
- E) 516

46. For all real numbers  $x$  and  $y$ , define  $x*y = x^2 - y^2$  , What is  $1001 * 999$  ?

- A) 500
- B) 1000
- C) 2000
- D) 3000
- E) 4000

- There are six mailboxes numbered 1, 2, 3, 4, 5, and 6 in front of an apartment.
- There is at least one mail in each box.
- The number of box and the number of mails in it are different. For example, there are not 3 mails in Box 3.
- The number of mails in the boxes may be equal or may be different.

47. The number of mails in Box 1 and Box 2 are the same. What is the least total number of mails in six boxes?

- A) 8
- B) 9
- C) 10
- D) 11
- E) 12

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48. The total number of mails in Box 2 and Box 4 is 5, and Box 4 and Box 5 is 7. How many mails are there in Box 5?

- A) 6
- B) 4
- C) 3
- D) 2
- E) 1

Use the information below to answer problems 47 & 48 !!!

49. For  $AB$  and  $ABC$  are two-digit and three-digit natural numbers respectively, such that  $AB \times 7 = ACB$ , What is  $A+B+C$  ?

- A) 5
- B) 6
- C) 7
- D) 8
- E) 9

50. Note that  $1001 : 7 = 143$ . When  $100110011001100110011001$  is divided by 7 what is the remainder?

- A) 3
- B) 0
- C) 1
- D) 2
- E) 4